



Uniform Coverage of Fibres over Open-contoured Freeform Structure Based on Arc-length Parameter

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Abstract

This article uses arc-length parameters for path planning to carry out robotic fibre placement (RFP) over open-contoured structures. This allows representing the initial path and offset points using an identical mathematical equation and computation by more simple arithmetic. With the help of classical differential geometry, the calculation of fiber-placing paths may be reduced to solution of initial-value problems of first-order ordinary differential equations in the parametric domain (parametrically defined mould surface) or in 3D space (an implicitly defined mould surface), thereby significantly improving on the existing methods. Compared with the conventional methods, the proposed method, besides its computational simplicity, has a better error control mechanism in computing the initial path and offset points. Numerical experiments are also carried out to demonstrate the feasibility of the new method in composite forming processes and also its potential application in computer numerical control (CNC) machining, surface trim, and other industrial practices.

Keywords: path planning; fibre placement; offset curve; uniform coverage

1 Introduction

Fibre-reinforced composites have found ever broader applications in aircraft and large carrier rocket manufacturing and other military and civil industries because of the advantages such as, in comparison with conventional materials, their higher strength-to-weight and stiffness-to-weight ratios, resistance to corrosion, ease to shape and tailor their structural configurations to meet the aerodynamical requirements by products. As an automatic and most preferential manufacturing technique for producing fibre-reinforced composite components, robotic fibre placement (RFP)^[1-8] must be based on the program control, in which the path

planning plays an important role, because it directly determines the success or failure of fibre placement as well as the properties of the products. Practices have proved that reasonable path coverage should adequately meet the surface geometric properties, the stress and strength requirements by produced components as well as simplifies computation. Moreover, the basic principle of the path coverage that has to be observe lies in minimizing gaps and overlaps between two neighboring paths. Offset from the initial path as the standard reference path may after all be an effective means to create such a uniform coverage, with which, Refs.[9-10] suggested the path planning over pipe-type mould surface, such as aircraft S-type inlet. In fact, there exist various alternatives for the initial path, for instance, an isoparametric curve^[11], a parametric function that maps onto the surface, a surface geodesic^[11], a surface-plane intersection curve, a curve that follows

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lines of principal stress^[12], a surface curve generated by projecting a space curve^[13] and so on. Among them, geodesic method is efficacious to generate an initial curve because it has no self-intersection for open-contoured surface, but it is deficient in necessitating solving initial-value problems of a system with second-order ordinary differential equations. A special planar intersection curve is developed based on the idea of average surface normal^[14], which, as an initial curve, is closer to a geodesic. In an attempt to improve on existing approaches^[13,15], this method begins with introducing arc-length parameters in constructing uniform coverage and then simplifies descriptive equations and computation of both initial path and path coverage. The new method is based on arc-length parameters and projection of straight line onto surface; it is termed as APPSL algorithm in short.

2 Mathematical Preliminaries

Let the mould surface be a differentiable parametric surface described by the following vector-valued function with two variables:

$$\mathbf{S}(u, v) = (x(u, v), y(u, v), z(u, v)), (u, v) \in D \subset \mathbf{R}^2 \quad (1)$$

where $x(u, v)$, $y(u, v)$, $z(u, v)$ are differentiable bivariate functions with two variables — surface parameters — of u and v , and D is the surface domain. Usually, Eq.(1) is called parameterization of the surface \mathbf{S} . The partial derivatives of the vector-valued function \mathbf{S} with respect to u and v are $\mathbf{S}_u(u, v)$, $\mathbf{S}_v(u, v)$, $(u, v) \in D$.

From the classical differential geometry, it is known that the vectors \mathbf{S}_u and \mathbf{S}_v are tangent to the surface \mathbf{S} at the point (u, v) , whereas the vector $\mathbf{S}_u \times \mathbf{S}_v$ is normal to the surface at the same point. The vector $\mathbf{N} = \mathbf{S}_u \times \mathbf{S}_v / |\mathbf{S}_u \times \mathbf{S}_v|$ is called the unit normal vector (or normal vector in short) of the surface \mathbf{S} at the corresponding point. Let the surface \mathbf{S} be regular, i.e. $\mathbf{S}_u \times \mathbf{S}_v \neq \mathbf{0}$ for any $(u, v) \in D$. A curve on the surface can be described by ensuing parametric equations in the parametric domain of the surface:

$$u = u(t), v = v(t) \quad t \in [a, b]$$

Its equation in \mathbf{R}^3 can be written into the vector-valued function:

$$\mathbf{P}(t) = \mathbf{S}(u(t), v(t)) \quad t \in [a, b] \quad (2)$$

By taking the derivative of Eq.(2) with respect to t , the following equation in the tangent space of the surface can be obtained:

$$\mathbf{P}'(t) = \mathbf{S}_u du/dt + \mathbf{S}_v dv/dt \quad t \in [a, b] \quad (3)$$

Now let the mould surface be represented by the implicit equation $f(x, y, z) = 0$. Obviously, implicit surfaces differ from the parametric surfaces in both appearance and expression. So do the curves on a parametric surface and on an implicit one. Here it is demanded that the function f must be continuous and differentiable. This means that the first partial derivatives $f_x = \partial f / \partial x$, $f_y = \partial f / \partial y$, $f_z = \partial f / \partial z$ must be continuous and all are not equal to zero simultaneously everywhere on the surface, that is, the surface is regular. The vector $\nabla f = (f_x, f_y, f_z)$ is called the gradient of the implicit surface at the point (x, y, z) . The vector $\mathbf{N} = \nabla f / |\nabla f|$ is the unit normal vector (or normal vector in short) of the implicit surface at the point (x, y, z) . A curve in parametric form on the implicit surface is expressed by $\bar{\mathbf{P}}(t) = (x(t), y(t), z(t))$. It is characterized by the following equation:

$$f(x(t), y(t), z(t)) = 0$$

By linearizing it, the following equation can be obtained:

$$f_x \frac{dx}{dt} + f_y \frac{dy}{dt} + f_z \frac{dz}{dt} = 0 \quad (4)$$

The gradient ∇f is perpendicular to the surface at the point (x, y, z) .

3 Initial Path Design with APPSI

In RFP, the choice of an initial curve, from which all subsequent paths will be determined, plays a critical role in ensuring uniform coverage of paths over the mould surface. As for the fibre placement to freeform surface structures, the attention should be centered on such an initial path planning where the initial path is created by the mould

surface intersecting a plane, i.e. projecting a straight line in a direction onto the mould surface.

Let $L = A_0 + Bt$ be a straight line perpendicular to the average normal vector $N_a^{[14]}$ of the mould surface; then project L along the vector N_a onto the mould surface, in other words, make the mould surface intersect a plane passing L and parallel to N_a , thereby acquiring a curve $P(s)$. Let s be its arc-length parameter, compute the curve with the technique of classical differential geometry (see Fig.1).

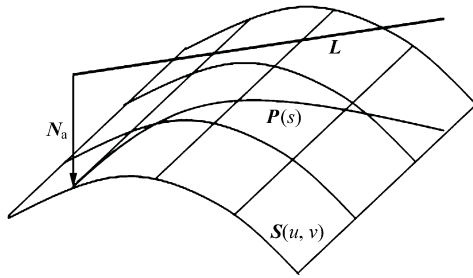


Fig.1 Initial path created by projecting L onto the mould surface.

It is obvious that

$$P'(s) \cdot (B \times N_a) = 0 \quad (5)$$

and

$$P'(s) \cdot N(s) = 0 \quad (6)$$

where $N(s)$ is the normal vector of the mould surface along the curve $P(s)$.

In addition, when s is an arc-length parameter, the following expression holds true:

$$(P'(s))^2 = 1 \quad (7)$$

From Eqs.(5), (6), (7) and Eq.(3), the following equations can be obtained:

$$\left. \begin{aligned} \frac{du}{ds} &= \frac{\pm S_v \cdot (B \times N_a)}{\sqrt{[(B \times N_a) \times N(s)]^2}} \\ \frac{dv}{ds} &= \frac{\mp S_u \cdot (B \times N_a)}{\sqrt{[(B \times N_a) \times N(s)]^2}} \end{aligned} \right\} \quad (8)$$

In order to completely determine system Eq.(8), the following initial-valued conditions on the surface domain D must be imposed.

$$u(0) = u_0, v(0) = v_0 \quad (9)$$

Once the systems Eq.(8) and Eq.(9) are solved by numerically integrating over u and v , the discrete

point set of the initial path can be obtained by substituting u and v into Eq.(2). By fitting the point set^[16-17], the parametric representation of the initial path can be found.

Particular attention should be paid to the combinations of signs in Eq.(8). Since s is a nonnegative parameter, there are only two possible pairs of sign combination: + with -, or - with +, which represent two opposite marching directions at the same start point (same as below).

If the mould surface is described in implicit form $f(x, y, z) = 0$, the intersection curve can achieve similar results:

$$Q'(s) = \left[\frac{dx}{ds} \quad \frac{dy}{ds} \quad \frac{dz}{ds} \right]^T = \frac{\pm \nabla f \times (B \times N_a)}{\sqrt{[\nabla f \times (B \times N_a)]^2}} \quad (10)$$

Imposing the following initial value conditions on Eq.(10)

$$x(0) = x_0, y(0) = y_0, z(0) = z_0 \quad (11)$$

results in an initial value problem of the first-order system with ordinary differential equations in 3D space. By numerically integrating it, the discrete point set of the initial path can be obtained.

4 Uniform Coverage Path

Let curve $P(t)$ be an initial path. To carry out uniform path coverage over the mould surface, the second path should be constructed by offsetting the initial path with a tow-width along the mould surface in the perpendicular direction. The third path can be obtained by offsetting the second path in the same manner. The process repeats until the required path coverage is generated.

Now, it comes to the way to offset a path on the mould surface (see Fig.2). Here, the path must be discretized into a point set $P = \{P(t_i), i = 1, 2, \dots, n\}$. At the point $P(t_i)$, compute its offset point in the following order: First, calculate the derivative vector $P'(t_i)$ of curve $P(t)$ at $P(t_i)$; second, construct a plane Π passing $P(t_i)$ with $P'(t_i)$ as its normal vector; third, compute the intersection curve $Q(s)$ of the mould surface and the plane Π (i.e. projecting a normal line of curve $P(t)$ at $P(t_i)$ onto the mould surface), assuming that the surface has been param-

eterized by the arc-length parameter. Then the point $\mathbf{Q}(s_{i0})$ on the intersection curve is the offset point of the point $\mathbf{P}(t_i)$.

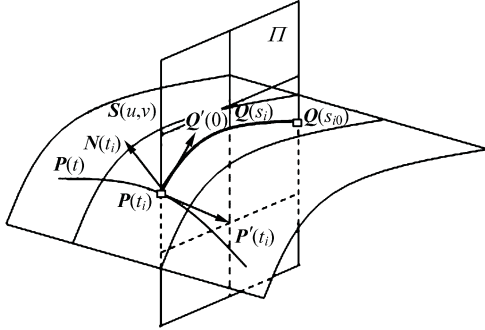


Fig.2 Offsetting point $\mathbf{P}(t_i)$ on a path to point $\mathbf{Q}(s_{i0})$ by a distance s_0 along surface/plane intersection curve.

As for calculation of the offset point $\mathbf{Q}(s_{i0})$, the same technique (APPSL) as described in Section 3 can be used to derive the differential equations of the intersection curve of the mould surface and the plane Π .

$$\left. \begin{aligned} \frac{du}{ds} &= \frac{\pm \mathbf{S}_v \cdot \mathbf{P}'(t_i)}{\sqrt{[\mathbf{P}'(t_i) \times \mathbf{N}(s)]^2}} \\ \frac{dv}{ds} &= \frac{\mp \mathbf{S}_v \cdot \mathbf{P}'(t_i)}{\sqrt{[\mathbf{P}'(t_i) \times \mathbf{N}(s)]^2}} \end{aligned} \right\} \quad (12)$$

In order to completely determine Eq.(12), the initial condition should be stipulated:

$$\bar{u}(0) = \bar{u}_0, \bar{v}(0) = \bar{v}_0 \quad (13)$$

where $\mathbf{P}(t_i) = \mathbf{S}(\bar{u}_0, \bar{v}_0)$. Once the Eq.(12) and Eq. (13) have been solved by numerical integrating over u and v , the offset point $\mathbf{Q}(s_{i0})$ on the intersection curve can be obtained by substituting the corresponding u and v into the equation of the mould surface. In this way, another point set $\mathbf{Q} = \{\mathbf{Q}(s_{i0}), i = 1, 2, \dots, n\}$ can be obtained, where $s_{i0} = s_0$ ($i = 1, 2, \dots, n$). Fitting these points leads to an offset path.

If the mould surface is described in implicit form $f(x, y, z) = 0$, similar results can be obtained for the intersection curve:

$$\mathbf{Q}'(s) = \left[\frac{dx}{ds} \frac{dy}{ds} \frac{dz}{ds} \right]^T = \frac{\pm \nabla f \times \mathbf{P}'(t_i)}{\sqrt{[\nabla f \times \mathbf{P}'(t_i)]^2}} \quad (14)$$

Imposing the following initial value conditions on Eq.(14):

$$\bar{x}(0) = \bar{x}_0, \bar{y}(0) = \bar{y}_0, \bar{z}(0) = \bar{z}_0 \quad (15)$$

results in an initial value problem of a first-order system with ordinary differential equations in 3D space. Numerically integrating it, the intersection curve and hence the offset point $\mathbf{Q}(s_{i0})$ can be computed. Sometimes, increasing n or extrapolating is needed in order to compute offset points or offset curve. For detailed exposition of RFP path extension (RPE) method, readers can refer to Ref.[15]. Fig.3 shows the flow chart about the whole process and steps of the proposed algorithm for RFP.

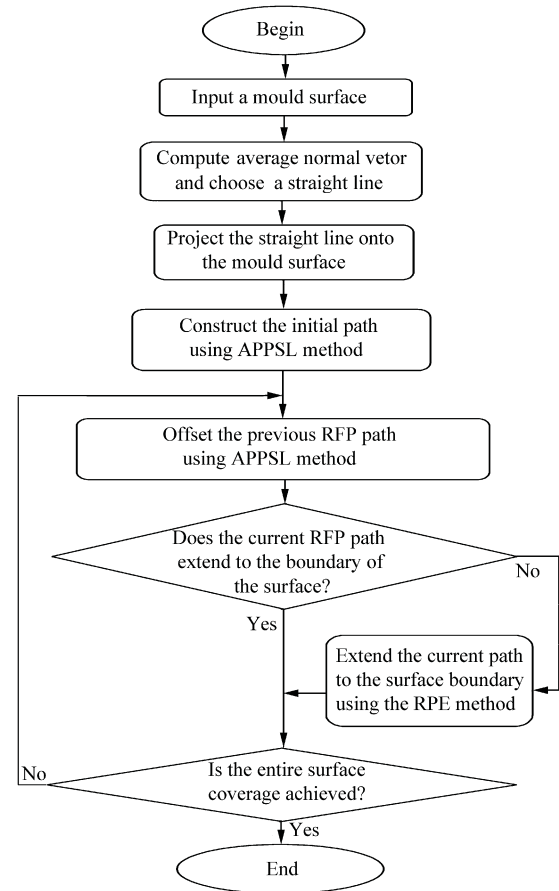


Fig.3 Algorithm steps of fibre uniform coverage over a mould surface for RFP.

5 Numerical Computation

The desired initial path and offset points for each specific case should be achieved by solving its corresponding initial value problem. Unfortunately, this article could not present analytical solutions to all systems of first-order ordinary differential equations (ODEs) excepting those involving some spe-

cial surfaces. In general, to deal with the most typical surfaces in computer-aided geometric design, the systems could be solved very efficiently by using standard numerical techniques. Once the initial values that can be obtained with the method^[18] have been determined, the first-order ODEs systems can be integrated numerically by means of methods^[19-20] with rapidity and reliability. For example, practice has proved it quite successful to use the ODEs solver function ODE45 of MATLAB^[19], which is based on an adaptive step-by-step technique combining 4th- and 5th-order Runge-Kutta methods for controlling the errors and the steps. In addition, these solvers provide users with good controls of tolerance^[21].

Generally speaking, the proposed method works well in addressing parametric or implicit surfaces, although some special cases should undergo careful analysis beforehand. For instance, in the case of a surface composed of several piecewise continuous patches, some kinds of continuity conditions must be stipulated to ensure the validity of the differential model in the neighborhood of each patch boundary.

In practical applications, the paths are needed to describe in the standard form such as B-spline or NURBS. However, the aforesaid numerical integration yields, an array of points in parametric domain and hence on the surface. Fortunately, using, for example, a cubic B-spline to interpolate those points on the surface creates a closed form of B-spline fitting paths either initial or offsetting. This time, the particular fitting method developed by F. E. Wolter^[16] or J. Qu^[17] fills the bill, for it affords good accuracy. Moreover, based on the method in Ref. [22], the B-spline approximation of the curve with array of points in the parametric domain can also be acquired.

6 Implementation Examples

The proposed method can be applied to any mould surface defined as implicit or parametric surface, including Bézier, B-spline and NURBS surface that are popular in CAGD and computer graphics.

For simplicity, only a Bézier surface and a sphere are taken to be representative of parametric and implicit surfaces to demonstrate the effectiveness of the method. Moreover, MATLAB6.5 is used to, especially, solve the differential equation to collect necessary data and then CATIA to generate the path coverage over the Bézier-type mould surface. Fig.4 and Fig.5 show application examples of robotic fibre placement developed by CATIA.

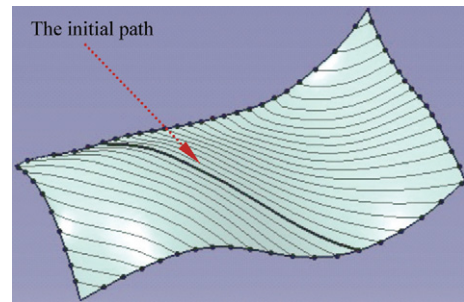


Fig.4 A uniform coverage of fibre paths over a Bézier-type mould surface.

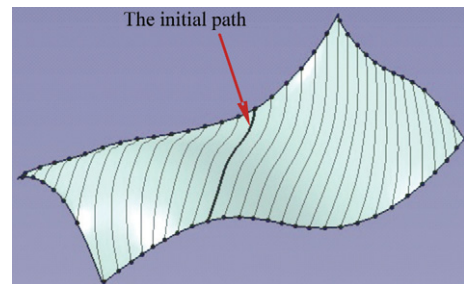


Fig.5 A uniform coverage of fibre paths with a different initial path over the same mould surface as in Fig.4.

Next, the computational efficiency is compared between the proposed IRP method and the SCAP method^[13] as follows:

Consider a Bézier surface, a special B-spline surface, as a mould surface, whose control points are given by the matrix

$$\begin{bmatrix} (-4, 4, -1) & (-2, 4, 1) & (2, 4, 1) & (4, 4, -1) \\ (-4, 2, 1) & (-2, 2, 3) & (2, 2, 3) & (4, 2, 1) \\ (-4, -2, 1) & (-2, -2, 3) & (2, -2, 3) & (4, -2, 1) \\ (-4, -4, -1) & (-2, -4, 1) & (2, -4, 1) & (4, -4, -1) \end{bmatrix}$$

Project the straight line $P(t) = (0, 0, 6) + (0, 1, 0)t$ in the direction $(0, 0, -1)$ onto the mould surface to acquire the initial path and then offset it to obtain the first range offset points. Having completed the

aforsaid process with both methods, the results are gathered in a table form (see Table 1).

Table 1 Efficiencies of two algorithms

Parameters	SCAR method	APPSL method
Assumed tow-width for offset	1	1
Number of data points in path	201	201
Marching step/ 10^{-2}	4	4
Average CPU time (10 times)/s	0.365	1.442

7 Conclusions

By choosing an arc-length parameter, a simpler path planning algorithm for RFP over open-contoured freeform structure is developed. The algorithm formulates the initial path and offset points as numerical solutions to the initial-value problems of systems with first-order ordinary differential equations. Moreover, it mainly considers uniform path coverage that ensures uniform lay-up of subsequent tows without gaps or overlaps. Unlike the method in Ref.[13], the proposed one theoretically could be reckoned accurate by neglecting errors in computation. The numerical analysis shows that it has an edge over existing plane-intersection and offset algorithms^[13,15] in simplifying computation and better error control. Examples show that the absolute error in computing initial path and offset points amounts to 10^{-10} , whereas, the absolute error in fitting these points to create paths achieves 10^{-7} . Compared with the method in Ref.[15], its most prominent advantages lie in that first, by adopting an arc-length parameter as a variable, users can easily increase or decrease the data points according to the curvature of mould surface in the tracing process as is required. Secondly, in contrast to Ref.[13], it allows users to compute the initial path and offset curves with the same equations thus greatly saving enormous amount of computational time. Thirdly, it reduces the workload in solving first-order ordinary differential equations to compute the initial path and offset points by adopting a quite ripe technique, the function ODE45 of MATLAB^[19], which is based on an adaptive step technique and has a mechanism for

controlling both relative and absolute errors. For more detailed information, readers can refer to any MATLAB textbook.

Some suggestions are put forward when a mould surface is constructed in piecewise manner. Future research should aim at creating such an initial RFP path that quantitatively reflects the geometric properties of a mould surface, such as normal curvature, curvature vector and so on at some discrete points, or follows the principle stress field of freeform structure.

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